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A FUZZY MEAN- COLOG PORTFOLIO SELECTION MODEL

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Abstract

A risk assess should persuade several properties to facilitate an investor's penchants. Risk is an asymmetric, relative, heteroskedastic, multidimensional concept that has to take into account asymptotic behaviour of returns, inter-temporal dependence, risk-time aggregation, and the impact of several economic phenomena that could influence an investor's preferences. We call uncertainty measure any increasing function of a positive functional defined on the space of random variables satisfying some specific properties. One example of an uncertainty measure is the Colog. We have extended the concept of mean-colog framework in fuzzy environment to propose a fuzzy Mean-Colog portfolio selection model. Numerical example is given for illustration.

Keywords:

Fuzzy number;
Fuzzy colog;
Possibilistic moment;
Stock portfolio selection

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1. Introduction

Although the foundation of modern mathematical models in economics can be traced back to L. Bachelier's (1900) dissertation on the theory of speculation in, without hesitation, the work of H. Markowitz (1952) in portfolio selection has been the most impact-making development in mathematical finance management. Since returns are uncertain in nature, the allocation of capital in different risky assets to minimize the risk and to maximize the return is the main concern of portfolio selection.

Scientific methodology advocates initially observing fiscal phenomena and then describing and characterizing it a propos the tools and the information available. However, some studies on portfolio theory do not apply this approach. In fact, often proposals for risk measures for portfolio theory are just applications of measures found in the statistics literature. However, some of these

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proposed measures do not always consider the range of investor attitudes towards risk. The main interest of investors is the consistency of a risk measure with their preferences.

There is debate in the literature on the right meaning of risk and uncertainty. Holton (2004) proposes that a definition of risk has to allow for two vital components of observed phenomena: exposure and uncertainty. Furthermore, all the acceptable tools available to cope with risk can model only the risk that is apparent. Accordingly, researchers can use only a working definition of risk. That is, it is possible to operationally identify only an investor's perception of risk. Attempts to quantify risk have led to the notion of a risk measure. A risk measure is a functional that assigns a numerical value which is interpreted as a loss. Risk is subjective because it is related to an investor's perception of exposure and uncertainty. It is significant to understand that since risk measures connect a solitary number, they cannot capture the entire information available.

A risk measure has been valued mainly because of its capacity of ordering investor preferences. Several researchers have said that the investors' preferences are exactingly dependent on the potential states of the returns (Karni (1985)). Theoretical results validate various instinctive portfolio selection approaches rooted in the safety-first rules as a decisive factor for decision-making under vagueness (Roy (1952), Tesler (1955), Bawa (1976, 1978), and Ortobelli and Rachev (2001)). It is well recognized that risk is an asymmetric concept related to downside outcomes. While measuring risk, upside and downside potential outcomes should be considered differently. Furthermore, a measure of uncertainty is not necessarily ample in measuring risk. The standard deviation includes both positive and negative deviations from the mean as a potential risk. Thus, in this case, outperformance relative to the mean is castigated just as much as underperformance. Balzer (1990, 2001) and Sortino and Satchell (2001) have proposed that investment risk might be measured by a functional of the difference among the investment return and a specific yardstick. In particular, the most celebrated and used benchmark approaches are based on coherent risk measures (see Szegö (2002, 2004)). As a matter of fact, the intuitive characteristics of investment risk, which are defined in a coherent risk measure, represent one of the most important aspects of the analysis by Artzner et al (1999). However, even if a coherent risk measure coherently prices risk, it cannot consider exhaustively all investment characteristics. The benchmark might itself be a random variable, such as a liability benchmark (such as an insurance product or defined benefit pension fund liabilities), the inflation rate or possibly inflation plus some safety margin, the risk-free rate of return, the bottom percentile of return, a sector index return, a budgeted return or other alternative investments.

The most popular measure used as a proxy for risk is the standard deviation. However, as demonstrated in several papers, the standard deviation cannot always be utilized as a measure of risk because it is a measure of uncertainty. Nevertheless, the two ideas of uncertainty and risk are connected. Generally, we refer to a generic risk measure considering either a proper risk measure or a measure of uncertainty according to the definition in Ortobelli et al. (2005). Measures of uncertainty (dispersion measures) can be introduced axiomatically (see Ortobelli (2001)). We call uncertainty measure any increasing function of a positive functional D defined on the space of random variables satisfying the following properties:

$D\ (X + C) \leq D\ (X)$ for all X and constraints $C \geq 0$.

$D(0) = 0$, and $D(aX) = aD(X)$ for all X and $a > 0$.

$D(X) \geq 0$ for all X, with $D(X) \geq 0$ for non-constant X.

According to these properties, positive additive shifts do not increase the uncertainty of the random variable X and the uncertainty measure D is equal to zero only if X is a constant. Therefore, we can say that the functional D measures the degree of uncertainty. One example of an uncertainty measure sensitive to additive shifts is the Colog, defined as:

$$Colog\ (X) = E(X\ logX) - E(X)\ E(logX).$$

This measure satisfies the above three properties and it is consistent with preferences of risk averse investors. That is, if all risk averse investors prefer the gross return X to Y, then $Colog\ (X) \leq Colog\ (Y)$.(see Giacometti and Ortobelli (2001)). Particular uncertainty measures are the deviation measures (see Rockafellar et al. (2005)) that satisfy property 1 as equality (i.e.,

$D\,(X + C) = D\,(X)$ for all X and constants C), properties 2, 3 and $D(X + Y) \leq D(X) + D(Y)$ for all X and Y (Rachev et al. (2005)).

In most of the research works on portfolio selection, the common assumptions are that the investor have enough historical data and that the situation of asset markets in future can be reflected with certainty by asset data in past. However, it cannot always be made with certainty. The usual feature of financial environment is uncertainty. Mostly, it is realized as risk uncertainty and is modeled by stochastic approaches. However, the term uncertainty has the second aspect-vagueness (imprecision or ambiguity) which can be modeled by fuzzy methodology. In this respect, to tackle the uncertainty in financial market, fuzzy, stochastic-fuzzy and fuzzy-stochastic methodologies are extensively used in portfolio modeling. By incurring fuzzy approaches quantitative analysis, qualitative analysis, experts' knowledge and investors' subjective opinions can be better integrated into a portfolio selection model. Authors like Konno and Suzuki (1995), Leon et al. (2002), Vercher (2007), Bhattacharyya et al. (2011) and others use fuzzy numbers to embody uncertain returns of the securities and they define the portfolio selection as a mathematical programming problem in order to select the best alternative. In possibilistic portfolio selection models, two types of approaches are noticed. The return of a security is considered either as a possibilistic variable or as a fuzzy number. In the later case, the possibilistic moments of the fuzzy numbers are considered. Possibilistic portfolio models integrate the past security data and experts' judgment to catch variations of stock markets more plausibly. Tanaka and Guo (1999) propose two kinds of portfolio selection models by utilizing fuzzy probabilities and exponential possibility distributions, respectively. Inuiguchi and Tanino (2000) introduce a possibilistic programming approach to the portfolio selection problem under the minimax regret criterion. Lai et al. (2002), Wang and Zhu (2002) and Giove et al. (2006) construct interval-programming models for portfolio selection. Ida (2004) investigates portfolio selection problem with interval and fuzzy coefficients, two kinds of efficient solutions are introduced: possibly efficient solution as an optimistic solution, necessity efficient solution as a pessimistic solution. Carlsson et al. (2002) introduce a possibilistic approach for selecting portfolios with the highest utility value under the assumption that the returns of assets are trapezoidal fuzzy numbers. Fang et al. (2006) propose a portfolio-rebalancing model with transaction costs based on fuzzy decision theory. Wang et al. (2005) and Zhang and Wang (2005) discuss the general weighted possibilistic portfolio selection problems. Moreover, Lacagnina and Pecorella (2006) develop a multistage stochastic soft constraints fuzzy program with recourse in order to capture both uncertainty and imprecision as well as to solve a portfolio management problem. Lin et al. (2005) propose a systematic approach by incorporating fuzzy set theory in conjunction with portfolio matrices to assist managers in reaching a better understanding of the overall competitiveness of their business portfolios. Huang (2007) presents two portfolio selection models with fuzzy returns by criteria of chance represented by credibility measure. Fei (2006) studies the optimal consumption and portfolio choice with ambiguity and anticipation. Zhang et al. (2007) assume that the rates of return of assets can be expressed by possibility distribution. They propose two types of portfolio selection models based on upper and lower possibilistic means and possibilistic variances and introduce the notions of lower and upper possibilistic efficient portfolios. Li and Xu (2007) deal with a possibilistic portfolio selection problem with interval center values. Parra et al. (2001) introduce vague goals for return rate, risk and liquidity based on expected intervals. Terol et al. (2006) formulate a fuzzy compromise programming to the mean-variance portfolio selection problem. Huang (2008) proposes a mean-semivariance model for describing the asymmetry of fuzzy returns. Huang (2008) extends the risk definition of variance and chance to a random fuzzy environment and formulates optimization models where security returns are fuzzy random variables.

In this paper we have proposed a fuzzy Mean-Colog portfolio selection model that maximizes the mean i.e., return of the portfolio and minimizes the Colog i.e., risk/uncertainty of the portfolio. To do so, we have extended the definition of Colog in fuzzy environment. Stock price data is used for the illustration and effectiveness of the proposed model.

2. Fuzzy Colog

As we already have discussed in the Introduction, for a random variable X, the uncertainty measure Colog is defined as $colog\ (X) = E(X\ logX) - E(X)\ E(logX)$.

Instead of a random variable, we consider X to be a fuzzy variable. To be specific, we consider X to be a fuzzy number and denote it by $\tilde{X}$.

We define the Colog of the fuzzy number $\tilde{X}$as

$$Co\log\left(\tilde{X}\right) = E\left(\tilde{X}\log\tilde{X}\right) - E\left(\tilde{X}\right)E\left(\log\tilde{X}\right).$$

Theorem 2.1 Let $\tilde{X} = (a, b, c)$ be a triangular fuzzy number. Then, the Colog of the fuzzy number $\tilde{X} = (a, b, c)$ is $\frac{1}{36}[(5a + 4b + c)\log a + (4a + 8b + 4c)\log b + (a + 4b + 5c)\log c]$

Proof

$$Co\log\left(\tilde{X}\right) = Co\log\left(a,\ \mathrm{b},\ \mathrm{c}\right)$$

$$= \mathrm{E}\left(\left(a,\ \mathrm{b},\ \mathrm{c}\right) * \log\left(a,\ \mathrm{b},\ \mathrm{c}\right)\right) - E\left(a,\ \mathrm{b},\ \mathrm{c}\right)E\left(\log\left(a,\ \mathrm{b},\ \mathrm{c}\right)\right)$$

$$= \mathrm{E}\left(\left(a,\ \mathrm{b},\ \mathrm{c}\right) * \left(\log a,\ \log\mathrm{b},\ \log\mathrm{c}\right)\right) - E\left(a,\ \mathrm{b},\ \mathrm{c}\right)E\left(\left(\log a,\ \log\mathrm{b},\ \log\mathrm{c}\right)\right)$$

$$\left[\mathrm{Bansal}\left(2011\right)\right].$$

Now,

$$\left(a,\ \mathrm{b},\ \mathrm{c}\right) * \left(\log a,\ \log\mathrm{b},\ \log\mathrm{c}\right)$$

$$= \left(a\log a, b\log b, c\log c\right)\left[\because a,\ b,\ c, \log a,\ \log b,\ \log c \geq 0\right].$$

Now, by possibility theory approach [Bhattacharyya and Kar (2011)], we find that the expectation of a triangular fuzzy number (a, b, c) is $\frac{a + 4b + c}{6}$. Then,

$$Co\log\left(\tilde{X}\right) = \mathrm{E}\left(\left(a,\ \mathrm{b},\ \mathrm{c}\right) * \left(\log a,\ \log\mathrm{b},\ \log\mathrm{c}\right)\right) - E\left(a,\ \mathrm{b},\ \mathrm{c}\right)E\left(\left(\log a,\ \log\mathrm{b},\ \log\mathrm{c}\right)\right)$$

$$= E\left(a\log a, b\log b, c\log c\right) - E\left(a,\ \mathrm{b},\ \mathrm{c}\right)E\left(\left(\log a,\ \log\mathrm{b},\ \log\mathrm{c}\right)\right)$$

$$= \left(\frac{a\log a + 4b\log b + c\log c}{6}\right) - \left(\frac{a + 4b + c}{6}\right)\left(\frac{\log a + 4\log b + \log c}{6}\right)$$

$$= \frac{1}{36}\left[6\left(a\log a + 4b\log b + c\log c\right) - \left(a + 4b + c\right)\left(\log a + 4\log b + \log c\right)\right]$$

$$= \frac{1}{36}\left[\left(5a + 4b + c\right)\log a + \left(4a + 8b + 4c\right)\log b + \left(a + 4b + 5c\right)\log c\right].$$

3. Model Formulation

Let for $i = 1, 2, \ldots, n$,

x_i = the portion of the total capital invested in security i;

$\widetilde{p_i}$ = fuzzy number representing the closing price of the ith security at present;

$\widetilde{p_i'}$ = fuzzy number representing the estimated closing price of the ith security for the next year;

d_i = fuzzy number representing the estimated dividend of the i^{th} security for the next year;

$\widetilde{r_i}$ = fuzzy number representing the return of the ith security = $\frac{\widetilde{p_i'} - \widetilde{p_i} + d_i}{\widetilde{p_i}}$

We propose the following return-risk portfolio selection model:

$$(3.1)\begin{cases} Minimize\ Colog[\tilde{r}_1x_1+\tilde{r}_2x_2+....+\tilde{r}_nx_n] \\ subject\ to \\ E[\tilde{r}_1x_1+\tilde{r}_2x_2+....+\tilde{r}_nx_n]\geq\alpha \\ \sum_{i=1}^{n}x_i=1, x_i\geq 0, i=1,2,...,n. \end{cases}$$

where the value of α will be specified by the investors according to their needs.

Theorem 3.1 Suppose $\tilde{r}_i=(a_i,b_i,c_i),\ i=1,2,\dots,n$ are independent triangular fuzzy numbers. Then the model (3.1) generates the model (3.2).

$$(3.2)\begin{cases} Minimize\ \frac{1}{36}[\ log\sum_{i=1}^{n}a_ix_i\sum_{i=1}^{n}(5a_i+4b_i+c_i)x_i+log\sum_{i=1}^{n}b_ix_i\sum_{i=1}^{n}(4a_i+8b_i+4c_i)x_i \\ \qquad +log\sum_{i=1}^{n}c_ix_i\sum_{i=1}^{n}(a_i+4b_i+5c_i)x_i\ ] \\ subject\ \text{to} \\ \frac{1}{6}\sum_{i=1}^{n}(a_i+4b_i+c_i)x_i\geq\alpha \\ \sum_{i=1}^{n}x_i=100, x_i\geq 0, i=1,2,...,n. \end{cases}$$

Proof Since $\tilde{r}_i=(a_i,b_i,c_i),\ i=1,2,\dots,n$ are triangular fuzzy numbers, by extension Principle of Zadeh it follows that

$$\tilde{r}_1x_1+\tilde{r}_2x_2+....+\tilde{r}_nx_n=\left(\sum_{i=1}^{n}a_ix_i\ ,\ \sum_{i=1}^{n}b_ix_i\ ,\ \sum_{i=1}^{n}c_ix_i\right),$$

which is also a fuzzy number. Combining this with the results obtained in example 2.7, we are with the theorem.

4. Numerical Example

We consider the return of five stock data as follows:

$$\tilde{r}_1=(0.40,\ 0.415,\ 0.45),$$
$$\tilde{r}_2=(0.45,\ 0.475,\ 0.49),$$
$$\tilde{r}_3=(0.22,\ 0.236,\ 0.24),$$
$$\tilde{r}_4=(0.52,\ 0.537,\ 0.55),$$
$$\tilde{r}_5=(0.26,\ 0.283,\ 0.30).$$

We solve the model (3.2) by the software LINGO for different values of α. The solutions obtained are shown in table 1.

Table 1: Portfolios with respect to different desired returns

α	x_1	x_2	x_3	x_4	x_5	Return	Colog
50	0	57.672	0	42.328	0	50	7026.845
45	0	90.251	9.749	0	0	45	6149.565
40	0	69.359	30.641	0	0	40	5291.914

It is clear from table 4.1 that when risk (Colog) is increasing, return (mean) is also increasing and vice-versa.

4. Conclusion

In this paper, we have introduced a fuzzy mean Colog portfolio selection model. The uncertainty measure Colog satisfies all the properties of an uncertainty measure. The numeric example clarify that the conflicting behavior of return and risk are kept in the proposed model.

In future, we will introduce some more constraints in the model to make it more realistic. We will also include the effect of transaction cost in the model. We will also use larger data sets for testing the effectiveness of the proposed model. Instead of the software LINGO, we will use genetic algorithm/ ant colony optimization algorithm to solve the model.

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